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Dimensional Crossover and Finite-Size Scaling below T_c

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Using the formalism developed in earlier work, dimensional crossover on a d-dimensional layered Ising-type system satisfying periodic boundary conditions and of size L is considered below $T_c(L)$, $T_c(L)$ being the critical temperature of the finite-size system. Effective critical exponents δ_{eff} and β_{eff} are shown explicitly to crossover between their d- and $(d-1)$-dimensional values for $\xi_L \to \infty$ in the limits $L/\xi_L \to \infty$ and $L/\xi_L \to 0$, respectively, ξ_L being the correlation length in the layers. Using an L-dependent renormalization group, the effective exponents are shown to satisfy natural generalizations of the standard scaling laws. In addition, L-dependent global scaling fields which span the entire crossover are defined and a scaling form of the equation of state in terms of them derived. All the above assertions are verified explicitly to one loop in perturbation theory, in particular effective exponents and a universal crossover equation of state are obtained and shown in the above asymptotic limits to be in good agreement with known results.

KEY WORDS: Equation of state; renormalization group; effective exponents; crossover scaling laws; finite size scaling; dimensional crossover.

1. INTRODUCTION

One of the most striking features of continuous phase transitions is the appearance of singularities, the singularities being associated with fixed points of the renormalization group (RG). Systems that possess more than one fixed point can exhibit crossover behavior between the various fixed points. This crossover behavior is important both theoretically and experimentally, but is difficult to treat. One can understand this intuitively in the following way: physical systems characteristically look very different at different "scales," exhibiting different effective degrees of freedom. A general

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discussion of crossover behavior in a field-theoretic RG context can be found in ref. 2.

Developing RGs that potentially offer full, global scaling information is not simple; see ref. 3 for some examples. The desire to make accessible another fixed point besides the isotropic one has often entailed the matching of asymptotic expansions around the anisotropic and isotropic fixed points[4] or the use of high-temperature expansions in conjunction with an ansatz for the scaling function.[5] Global RGs can be found most simply using field-theoretic methods. Here though one encounters the commonly held prejudice that renormalization is entirely due to short-distance singularities. If one holds to this view, then it is not sensible to develop RGs that depend on relevant "infrared" scales. Implementing the point of view that renormalization can depend on important IR scales, a small number of crossovers have been treated in a more appropriate manner, e.g., crossover at a bicritical point,[6] crossover in uniaxial dipolar ferromagnets,[7] and dimensional crossover.[1] It is with the latter that we will be exclusively concerned and in particuar with the extension of the techniques of ref. 1 to systems below T_c.

Dimensional crossover has been chiefly addressed in the context of finite-size scaling.[8] A great deal of effort has been put into finite-size scaling and finite-size effects in the context of lattice simulations (see, for instance, ref. 9 for an early review, and ref. 10 for recent results, where interestingly the concept of an effective dimensionality in a purely numerical context arises). In most work on the RG applied in the context of finite-size scaling[11,12] it has been a "bulk" RG, which is independent of the finite-size scale L, that has been used. Such an RG has proved incapable of furnishing finite-size scaling functions and dimensional crossover information except when supplemented by further nonperturbative information.[15] In ref. 15, systems were considered that do not exhibit a true crossover in the sense that the finite system possesses only one fixed point—the "bulk" fixed point. Finite-size scaling functions were obtained by, in the case of a totally finite system, treating the zero mode of the theory exactly, this exact treatment representing the extra nonperturbative information beyond the RG alluded to above. Some results in exact models[13] have been derived for crossover functions between two nontrivial fixed points, and in ref. 14 the dimensional crossover between mean-field theory and a nontrivial fixed point using the ε-expansion was considered. The latter is useless, however, for treating the case of crossover between two nontrivial fixed points. In ref. 1 a formalism was developed that can treat finite-size systems that either do or do not possess more than one nontrivial fixed point, though the emphasis was completely on the former. The essence of the methodology is an L-dependent RG implemented in the spirit that the "true" effective degrees of freedom of the system are L dependent.

Before outlining the plan of the paper let us mention the advantages of our approach and elucidate what we believe to be novel in the paper. The advantages of the formulation from an RG point of view are: (i) it emphasizes ξ_L, which appears naturally in both experiment and numerical simulations; (ii) it allows for a systematic perturbative calculation of finite-size scaling functions; (iii) it is capable of treating systems with more than one nontrivial fixed point within the one globally defined, perturbatively controllable RG. The things we believe to be novel in the paper are the following: (i) the formal derivation of scaling forms for correlation functions in terms of ξ_L and effective critical exponents, in particular the crossover equation of state; (ii) a demonstration using an L-dependent RG that the effective exponents obey natural analogs of the scaling laws; (iii) the derivation of explicit perturbative expressions for the effective exponents β_{eff} and δ_{eff}, between 3 and 2 dimensions and 4 and 3, which asymptotically are in good agreement with known results (consequently we verify in perturbation theory the exponent relations; the only other theoretical work we are aware of with analogous considerations is ref. 16 in the context of analytic corrections to scaling in planar Ising models); (iv) the derivation of a perturbative expression for the crossover equation of state.

The plan of this paper is as follows. In Section 2 the renormalization group equation (RGE) below T_c is deduced and the scaling form of vertex functions throughout the crossover are discussed. We also discuss the concept of effective dimensionality.[1,2] In Section 3, the effective exponent laws involving δ_{eff} and β_{eff} are derived and in Section 4 the one-loop universal scaling form of the crossover equation of state is calculated. We present the crossover coexistence surface in graphical form in Fig. 1. Figure 2 presents the magnetization as a function of temperature for different L on the coexistence curve. It compares favorably with numerical results of Binder.[9] Section 5 gives the one-loop results for the floating fixed-point effective exponents β^*_{eff}, δ^*_{eff}, and d^*_{eff}. These are seen to cross over between the perturbative expressions for the associated limiting fixed dimension exponents. β^*_{eff} is presented in graphical form in Fig. 3 for three- and four-dimensional layered geometries. The corresponding exponents δ^*_{eff} and d^*_{eff} are presented in Fig. 4. Section 6 is reserved for conjectures and conclusions.

2. SCALING BELOW T_c

The theory that we use as a prototypical example throughout this paper is an Ising-type system described by the Landau–Ginzburg–Wilson Hamiltonian

$$\mathscr{H} = \frac{1}{2}(\nabla\varphi_B)^2 + \frac{1}{2}m_B^2\varphi_B^2 + \frac{1}{2}t_B\varphi_B^2 + \frac{\lambda_B}{4!}\varphi_B^4 - H_B\varphi_B \tag{1}$$

on $S^1 \times \mathbb{R}^{d-1}$, i.e., a layered geometry with periodic boundary conditions with $(d-1)$-dimensional layers and of total thickness L. We restrict our considerations to $3 \leqslant d \leqslant 4$. We know that in the broken phase of an Ising-type system the correlation functions are functions of the magnetization density $M = \langle \varphi \rangle$ of the system which we assume to be homogeneous.

The system is renormalized by locating the critical point at $t = 0$ and subject to the following conditions for the renormalization of the coupling and the composite operator φ^2 (we follow the notation of ref. 17 and postpone discussion of wavefunction renormalization for the moment)

$$\Gamma^{(4)}(k = 0, t = \kappa^2, \lambda, L, \kappa) = \lambda \tag{2}$$

$$\Gamma^{(2,1)}(k = 0, t = \kappa^2, \lambda, L, \kappa) = 1 \tag{3}$$

Our notation is that $\mathbf{k}$ is the momentum in the layers and k includes the discrete momentum perpendicular to the layers. From (2) we also define a dimensionless coupling $\bar{\lambda} = \lambda \kappa^{(d-4)}$. Obviously as these normalization conditions are L dependent, the consequent renormalized parameters are implicitly L dependent. From the κ independence of $\Gamma_B^{(N)}$ we find the RGE for $\Gamma^{(N)}$

$$\left\{\kappa \frac{\partial}{\partial \kappa} + \beta(\bar{\lambda}, L\kappa) \frac{\partial}{\partial \bar{\lambda}} + \gamma_{\varphi^2}(\bar{\lambda}, L\kappa)\, t \frac{\partial}{\partial t} - \frac{1}{2} \gamma_\varphi(\bar{\lambda}, L\kappa) \left[N + M \frac{\partial}{\partial M}\right]\right\} \Gamma^{(N)} = 0 \tag{4}$$

Its solution can be found by the method of characteristics

$$\Gamma^{(N)}(t, M, \bar{\lambda}, L, \kappa) = \exp\left[-\frac{N}{2} \int_\kappa^{\rho\kappa} \gamma_\varphi(x, Lx) \frac{dx}{x}\right] \Gamma^{(N)}(t(\rho), M(\rho), \bar{\lambda}(\rho), L, \kappa\rho) \tag{5}$$

where the running variables $t(\rho)$, $M(\rho)$, and $\bar{\lambda}(\rho)$ satisfy the characteristic equations

$$\rho \frac{dt(\rho)}{d\rho} = \gamma_{\varphi^2}(\bar{\lambda}(\rho), L\kappa\rho)\, t(\rho) \tag{6}$$

$$\rho \frac{dM(\rho)}{d\rho} = -\frac{1}{2} \gamma_\varphi(\bar{\lambda}(\rho), L\kappa\rho)\, M(\rho) \tag{7}$$

$$\rho \frac{d\bar{\lambda}(\rho)}{d\rho} = \beta(\bar{\lambda}(\rho), L\kappa\rho) \tag{8}$$

We can also rewrite (8) after a change of variable back to the dimensionful coupling λ as

$$\rho \frac{d\lambda(\rho)}{d\rho} = \gamma_\lambda(\lambda(\rho), L\kappa\rho)\, \lambda(\rho) \tag{9}$$

The left-hand side of (5) cannot be evaluated perturbatively at $t=0$, due to infrared divergences. This problem is surmounted by proceeding analogously to a system without crossover. The arbitrariness of ρ is utilized by trying to choose it so that the system is kept away from the infrared dangerous region for any value of L. Some possible conditions one might envision using to determine ρ are: $t(\rho)=(\kappa\rho)^2$, $1/2\lambda(\rho)M^2(\rho)=\rho^2\kappa^2$, or $t(\rho)+1/2\lambda(\rho)M^2(\rho)=\rho^2\kappa^2$. The usual condition $M(\rho)=(\rho\kappa)^{d/2-1}$ is inappropriate for the crossover problem and will be discussed further at the end of Section 5.

We now turn our attention to wavefunction and $\Gamma^{(2)}$ renormalization. Consider the following sets of normalization conditions

$$\begin{aligned} &\Gamma^{(2)}(k=0, t(\rho), M(\rho)=0, \lambda(\rho), L, \kappa\rho)=t(\rho) \\ &\frac{\partial}{\partial \mathbf{k}^2}\Gamma^{(2)}(k, t(\rho), M(\rho)=0, \lambda(\rho), L, \kappa\rho)\bigg|_{k=0}=1 \end{aligned} \tag{10}$$

$$\begin{aligned} &\Gamma^{(2)}(k=0, t(\rho)=0, M(\rho), \lambda(\rho), L, \kappa\rho)=\frac{\lambda(\rho)}{2}M^2(\rho) \\ &\frac{\partial}{\partial \mathbf{k}^2}\Gamma^{(2)}(k, t(\rho)=0, M(\rho), \lambda(\rho), L, \kappa\rho)\bigg|_{k=0}=1 \end{aligned} \tag{11}$$

$$\begin{aligned} &\Gamma^{(2)}(k=0, t(\rho), M(\rho), \lambda(\rho), L, \kappa\rho)=t(\rho)+\frac{\lambda(\rho)}{2}M^2(\rho) \\ &\frac{\partial}{\partial \mathbf{k}^2}\Gamma^{(2)}(k, t(\rho), M(\rho), \lambda(\rho), L, \kappa\rho)\bigg|_{k=0}=1 \end{aligned} \tag{12}$$

From the definition of the physical correlation length in the layers

$$\xi_L^2 = \frac{\int d^dx\, \mathbf{x}^2 G_L^{(2)}(x,0)}{2d\int d^dx\, G_L^{(2)}(x,0)} \tag{13}$$

where $\mathbf{x}^2$ is the distance squared in the layers, one sees that conditions (10) imply $t(\rho)=\rho^2\kappa^2=\xi_{Lt}^{-2}$, where ξ_{Lt} is the correlation length in the finite-size system when $M=0$. With (11), $1/2\lambda(\rho)M^2(\rho)=\rho^2\kappa^2=\xi_{LM}^{-2}$, where ξ_{LM} is the correlation length in the finite-size system when $T=T_c(L)$. With (12), $t(\rho)+1/2\lambda(\rho)\,M^2(\rho)=\rho^2\kappa^2=\xi_{LMt}^{-2}$, where ξ_{LMt} is the generic correlation length in the finite size system. The correlation lengths ξ_{Lt}, ξ_{LM}, and ξ_{LMt} are all nonlinear scaling fields which are capable of interpolating between the d- and $(d-1)$-dimensional fixed points of the system for $\xi \to \infty$ in the limits $L/\xi \to \infty$ and $L/\xi \to 0$, respectively.

To see the explicit crossover between the d- and $(d-1)$-dimensional fixed points we glean some results from ref. 1. With the normalization condition (2) one finds to one loop

$$\rho \frac{d\bar{\lambda}}{d\rho} = -(4-d)\bar{\lambda} + \frac{3\bar{\lambda}^2}{L\kappa\rho}(4\pi)^{(1-d)/2}\,\Gamma\left(\frac{7-d}{2}\right) \times \sum_{n=-\infty}^{\infty}\left(1+\frac{4\pi^2n^2}{L^2\kappa^2\rho^2}\right)^{(d-1)/2} + O(\bar{\lambda}^3) \tag{14}$$

As $L\kappa\rho \to \infty$, $\rho \to 0$ this displays the normal d-dimensional fixed point. When $L\kappa\rho \to 0$ the natural coupling constant is $\bar{\lambda}/\kappa\rho L = u$ and for fixed L, u runs to the $(d-1)$-dimensional fixed point. Naturally such a change of variables cannot affect the physics, it merely makes the $(d-1)$-dimensional fixed point look familiar. One can take the solution of (14) as the "small" parameter with respect to which perturbation theory is ordered. In bulk critical phenomena one captures the dominant physics by expanding around a fixed point. In a crossover, however, there is more than one, hence corrections to scaling around one fixed point become very large when one approaches the other. We emphasize that in our formalism such corrections are computable. It would be advantageous, however, to mimic the standard formalism as much as possible by keeping corrections to scaling small. Consider then the change of variables $h = a_1\bar{\lambda}$, where a_1 is the coefficient of the $O(\bar{\lambda}^2)$ term in (14). One finds

$$\kappa\frac{dh}{d\kappa} = \beta(h) = -\varepsilon(L\kappa)h + h^2 + O(h^3) \tag{15}$$

where

$$\begin{aligned}\varepsilon(L\kappa) &= 4-d-\frac{d\ln a_1}{d\ln\kappa}\\ &= 5-d-(7-d)\frac{\sum_{n=-\infty}^{\infty}(4\pi^2n^2/L^2\kappa^2)(1+(4\pi^2n^2/L^2\kappa^2))^{(d-9)/2}}{\sum_{n=-\infty}^{\infty}(1+(4\pi^2n^2/L^2\kappa^2))^{(d-7)/2}}\end{aligned} \tag{16}$$

Setting $\beta(h)=0$ yields to lowest order

$$h^* = \varepsilon(L\kappa) + O(\varepsilon^2(L\kappa)) \tag{17}$$

We term h^* a floating fixed point.[1,2,18] Its importance is twofold. First, corrections to scaling around the floating fixed point are small. Second, it is, like a conventional fixed point, found from an algebraic property of the β function—its zeros. This is obviously computationally much simpler than having to solve a differential equation. The difference between using the solutions of (14) or the floating fixed point corresponds to slowly varying factors which are redefinitions of the L-dependent crossover variables such as t. For the case at hand h^* varies between $4-d$ and $5-d$. It is clear that the floating fixed point is not necessarily numerically small. In order to achieve accurate estimates of physical quantities one would in principle wish to work to higher order and attempt some resummation procedure analogous to what is done with the fixed dimension expansion.[19] Apart from lengthy calculation there is absolutely nothing to prevent this being done using the present techniques for $d<4$. Although we will restrict attention to one-loop results herein, two-loop results for $T>T_c(L)$ have been calculated in refs. 18 and 1. One also knows by experience that one-loop results are better than mean-field theory and that two-loop results are in fact quite often close to numerical and experimental results.

One can think of $\varepsilon(L\kappa)$ as being a measure of the deviation from four of the effective dimensionality of the system. More generally one can define an effective dimensionality of the system. More generally one can define an effective dimensionality[4] d_{eff} via the relation

$$\frac{d \ln \Gamma^{(4)}}{d \ln |t|} = (4 - d_{\text{eff}} - 2\eta_{\text{eff}})\,\nu_{\text{eff}} \tag{18}$$

where $\nu_{\text{eff}} = (2-\gamma_{\varphi^2})^{-1}$ and $\eta_{\text{eff}} = \gamma_\varphi$ are the anomalous dimensions of φ^2 and φ across the crossover. For $k=0$, $d_{\text{eff}} = 4-\gamma_\lambda$. γ_λ is related to the anomalous dimension of the dimensionful coupling constant and satisfies (9). As $\gamma_\lambda = \varepsilon(L\kappa) + \beta(h)/h$, one finds $d_{\text{eff}} = d - \beta(\bar{\lambda})/\bar{\lambda}$. Clearly d_{eff} interpolates between d and $d-1$ as h varies from the bulk to the reduced fixed points. In line with the simpler notion of a floating fixed point one can define a floating d_{eff}, d^*_{eff}, as $d^*_{\text{eff}} = 4 - \gamma^*_\lambda$. The d^*_{eff} also interpolates between d and $d-1$ and therefore captures the essence of the crossover, the difference between d_{eff} and d^*_{eff} being a slowly varying correction to scaling throughout the crossover. One can also define effective critical exponents ν^*_{eff} and η^*_{eff} with respect to the floating fixed point, i.e., $\nu^*_{\text{eff}} = \nu_{\text{eff}}(h=h^*)$ and $\eta^*_{\text{eff}} = \eta_{\text{eff}}(h=h^*)$.

[4] An analogous quantity was found by A. Bray in the context of the spherical model (private communication).

Now, consider (5) with the conditions (10). First, solving the characteristic equations (7) and (8) and expanding around the floating fixed point $h = h^*$ yields

$$\frac{\lambda(\rho)\, M^2(\rho)}{\rho^2 \kappa^2} = M^2 \exp\left[-\int_\kappa^{\xi_{Lt}^{-1}} (d^*_{\text{eff}} - 2 + \eta^*_{\text{eff}}) \frac{dx}{x}\right] \tag{19}$$

Substituting back into (5) and using dimensional analysis gives

$$\Gamma^{(N)} = \xi_{Lt}^{Nd/2 - N - d} \exp\left(-\frac{N}{2}\int_\kappa^{\xi_{Lt}^{-1}} \eta^*_{\text{eff}} \frac{dx}{x}\right) \times \mathscr{F}_t^{(N)}\left(M^2 \exp\left[-\int_\kappa^{\xi_{Lt}^{-1}} (d^*_{\text{eff}} - 2 + \eta^*_{\text{eff}}) \frac{dx}{x}\right], \frac{L}{\xi_{Lt}}\right) \tag{20}$$

where $\mathscr{F}_t^{(N)}$ is a universal function. So, if $\Gamma^{(N)}$ is measured in units of ξ_{Lt} we see that the scaling functions are functions of two nonlinear scaling variables L/ξ_{Lt} and

$$M^2 \exp\left[-\int_\kappa^{\xi_{Lt}^{-1}} (d^*_{\text{eff}} - 2 + \eta^*_{\text{eff}}) \frac{dx}{x}\right]$$

With the conditions (11) instead of (10) one finds

$$\Gamma^{(N)} = \xi_{LM}^{Nd/2 - N - d} \exp\left(-\frac{N}{2}\int_\kappa^{\xi_{LM}^{-1}} \eta^*_{\text{eff}} \frac{dx}{x}\right) \mathscr{F}_M^{(N)}\left(t \exp\left(-\int_\kappa^{\xi_{LM}^{-1}} \frac{1}{\nu^*_{\text{eff}}} \frac{dx}{x}\right), \frac{L}{\xi_{LM}}\right) \tag{21}$$

where $\mathscr{F}_M^{(N)}$ is also a universal function. For $\Gamma^{(N)}$ measured in units of ξ_{LM} these scaling functions are functions of the two scaling fields L/ξ_{LM} and

$$t \exp\left(-\int_\kappa^{\xi_{LM}^{-1}} \frac{1}{\nu^*_{\text{eff}}} \frac{dx}{x}\right)$$

The correlation length ξ_{Lt} interpolates between $t^{-\nu_b}$ and $t^{-\nu_r}$ for $\xi_{Lt} \to \infty$ in the limits $L/\xi_{Lt} \to \infty$ and $L/\xi_{Lt} \to 0$, respectively, where ν_b and ν_r are the bulk and reduced correlation length exponents. Note that all the above scaling fields are globally valid in the sense that they capture both the d- and $(d-1)$-dimensional fixed points. We could also have written down scaling functions $\mathscr{F}_{Mt}$ which would be functions of ξ_{LMt} which is also a good scaling field for the crossover.

3. SCALING LAWS

In the previous section we investigated the scaling form of vertex functions below T_c in terms of two scaling fields ξ_{Lt} and ξ_{LM}. In this section we would like to proceed further with a general scaling formulation, examining what happens to scaling laws for the crossover. In particular let us consider the crossover equation of state. From the noncrossover equation of state $H = M^\delta f(tM^{-1/\beta})$ it is natural to define effective critical exponents for the crossover

$$\delta_{\text{eff}} = \left.\frac{d \ln H}{d \ln M}\right|_{t=0} \qquad \text{and} \qquad \beta_{\text{eff}} = \frac{d \ln M}{d \ln |t|}$$

the latter being defined on the crossover coexistence curve.

When $T = T_c(L)$, i.e., $t = t(\rho) = 0$, we impose the normalization condition

$$H\left(\frac{\lambda(\rho)\, M^2(\rho)}{2\rho^2\kappa^2} = 1, h(\rho), L\kappa\rho\right) = \frac{\lambda(\rho)\, M^3(\rho)}{6(\rho\kappa)^{d/2+1}} \tag{22}$$

This condition is consistent with the normalization condition on $\Gamma^{(2)}$ and motivated by the mean-field theory case. With this normalization condition one finds

$$\frac{d \ln H}{d \ln M} = \left(\frac{d}{2} + 1 - \frac{1}{2}\gamma_\phi - \frac{1}{2}\frac{\beta(\bar{\lambda}(\rho))}{\bar{\lambda}(\rho)}\right)\frac{d \ln \rho}{d \ln M} \tag{23}$$

where the characteristic equation for $\bar{\lambda}(\rho)$ has been used. From Section 2, recall that $\beta(\bar{\lambda})/\bar{\lambda} = d - d_{\text{eff}}$, hence with the condition $\lambda(\rho)\, M^2(\rho)/2 = \rho^2\kappa^2$, one finds

$$\frac{d \ln \rho}{d \ln M} = \frac{2}{(d_{\text{eff}} - 2 + \gamma_\phi)}$$

Substituting back into (23) gives

$$\frac{d \ln H}{d \ln M} = \delta_{\text{eff}} = \frac{d_{\text{eff}} + 2 - \eta_{\text{eff}}}{d_{\text{eff}} - 2 + \eta_{\text{eff}}} \tag{24}$$

Now let us turn our attention to the relationship between M and t on the coexistence curve. Imposing the normalization condition

$$M(|t(\rho)| = \rho^2\kappa^2, \bar{\lambda}(\rho), L\kappa\rho) = \left[\frac{6|t(\rho)|}{\bar{\lambda}(\rho)\, \rho^2\kappa^2}\right]^{1/2} \tag{25}$$

which again corresponds to imposing the mean-field condition at the normalization point, and requires only a finite renormalization of $G^{(1)}$, and once again using the characteristic equation for $\bar{\lambda}(\rho)$, one finds

$$\frac{d \ln M}{d \ln |t|} = \frac{1}{2}\left(d - 2 + \gamma_\phi - \frac{\beta(\bar{\lambda}(\rho))}{\bar{\lambda}(\rho)}\right)\frac{d \ln \rho}{d \ln |t|} \tag{26}$$

With the condition $|t(\rho)| = \rho^2\kappa^2$ one has $d \ln \rho / d \ln |t| = \nu_{\text{eff}}$. Substituting into (26) gives

$$\frac{d \ln M}{d \ln |t|} = \beta_{\text{eff}} = \frac{\nu_{\text{eff}}}{2}(d_{\text{eff}} - 2 + \eta_{\text{eff}}) \tag{27}$$

Thus we get the very interesting result that natural analogs of the conventional scaling laws are obeyed throughout the entire crossover. What this implies is that there is a generalization of universality which applies across the crossover in the sense that knowledge of γ_ϕ and γ_{ϕ^2} are sufficient to determine the entire crossover along with one more function d_{eff}. Knowledge of d_{eff} is equivalent to knowledge of γ_λ. In other words, in contradistinction to the standard noncrossover problem, where γ_λ merely represents slowly varying corrections to scaling, here one requires γ_λ to obtain full knowledge of the crossover, i.e., the leading irrelevant operator is playing a significant role. It is also interesting that effective exponents defined with respect to the floating fixed point also obey scaling laws, explicitly

$$\delta^*_{\text{eff}} = \frac{d^*_{\text{eff}} + 2 - \eta^*_{\text{eff}}}{d^*_{\text{eff}} - 2 + \eta^*_{\text{eff}}} \qquad \text{and} \qquad \beta^*_{\text{eff}} = \frac{\nu^*_{\text{eff}}}{2}(d^*_{\text{eff}} - 2 + \eta^*_{\text{eff}})$$

The difference between a floating fixed-point and running coupling result amounts to no more than a redefinition of ones crossover variables by slowly varying nonsingular corrections to scaling across the crossover. In other words, the floating fixed point captures the "universal" part of the crossover.

Having introduced the effective exponents δ_{eff} and β_{eff} we can return to the considerations of Section 3 and write the scaling forms in a slightly different way. Consider (20) and (21), first (20). The integrals in (20) are from an initial to a final inverse correlation length, having used the relation $\rho^2\kappa^2 = \xi_{Lt}^{-1}$, hence we can change variables using the definition of ν_{eff}, i.e., $d\rho/\rho = -d\xi_{Lt}/\xi_{Lt} = \nu_{\text{eff}}\, dt/t$, to find

$$\Gamma^{(N)} = \exp\left[\int_1^t \left(N + d - \frac{N}{2}(d + \eta_{\text{eff}})\right)\nu_{\text{eff}}\frac{dt'}{t'}\right] \times \mathscr{F}_t^{(N)}\left(M \exp\left(-\int_1^t \beta_{\text{eff}}\frac{dt'}{t'}\right), L \exp\left(\int_1^t \nu_{\text{eff}}\frac{dt'}{t'}\right)\right) \tag{28}$$

The two scaling fields entering the scaling function, in terms of $T-T_c(L)$, are

$$M \exp\left(-\int_1^t \beta_{\text{eff}} \frac{dt'}{t'}\right) \qquad \text{and} \qquad L \exp\left(\int_1^t \nu_{\text{eff}} \frac{dt'}{t'}\right)$$

Now consider (21). Using the condition fixing ρ in terms of M, we can change variables

$$\frac{d\rho}{\rho} = -\frac{d\xi_{LM}}{\xi_{LM}} = \frac{2}{d_{\text{eff}}-2+\eta_{\text{eff}}} \frac{dM}{M}$$

to find

$$\Gamma^{(N)} = \exp\left[\int_1^M \left(N+d-\frac{N}{2}(d+\eta_{\text{eff}})\right)\frac{\nu_{\text{eff}}}{\beta_{\text{eff}}}\frac{dM'}{M'}\right]$$
$$\times \mathscr{F}_M^{(N)}\left(t \exp\left(-\int_1^M \frac{1}{\beta_{\text{eff}}}\frac{dM'}{M'}\right), L \exp\left(\int_1^M \frac{\nu_{\text{eff}}}{\beta_{\text{eff}}}\frac{dM'}{M'}\right)\right) \tag{29}$$

The equation of state in both cases is found simply by putting $N=1$.

Now, from the perturbative results, as we shall see in the next section, with the condition $1/2\lambda(\rho)\, M^2(\rho) = \rho^2\kappa^2$ one can extract a factor $[2/\bar{\lambda}(\rho)]^{1/2}$ from $\mathscr{F}_M^{(1)}$, the remainder of $\mathscr{F}_M^{(1)}$ being a polynomial expansion in $\bar{\lambda}(\rho)$ [or $h(\rho)$]. With

$$\bar{\lambda}(\rho) = \bar{\lambda} \exp\left(\int_1^M \left(\frac{2(d-d_{\text{eff}})}{d_{\text{eff}}-2+\eta_{\text{eff}}}\right)\frac{dM'}{M'}\right) \tag{30}$$

one obtains for the scaling form of the equation of state

$$H = \exp\left(\int_1^M \delta_{\text{eff}} \frac{dM'}{M'}\right) \mathscr{G}\left(t \exp\left(-\int_1^M \frac{1}{\beta_{\text{eff}}}\frac{dM'}{M'}\right), L \exp\left(\int_1^M \frac{\nu_{\text{eff}}}{\beta_{\text{eff}}}\frac{dM'}{M'}\right)\right) \tag{31}$$

in terms of the two scaling fields

$$x = t \exp\left(-\int_1^M \frac{1}{\beta_{\text{eff}}}\frac{dM'}{M'}\right) \qquad \text{and} \qquad y = L \exp\left(-\int_1^M \frac{\nu_{\text{eff}}}{\beta_{\text{eff}}}\frac{dM'}{M'}\right)$$

For $t=0$,

$$\mathscr{G} = 1 \qquad \text{and} \qquad H = \exp\left(\int_1^M \delta_{\text{eff}} \frac{dM'}{M}\right)$$

recovering (24). For $H=0$ the equation of state is given by $\mathscr{G}(x, y)=0$, which yields a coexistence curve $x=g(y)$, hence

$$t=g\left(L\exp\left(-\int_1^M \frac{\nu_{\text{eff}}}{\beta_{\text{eff}}}\frac{dM'}{M'}\right)\right)\exp\left(\int_1^M \frac{1}{\beta_{\text{eff}}}\frac{dM'}{M'}\right) \tag{32}$$

In order that we reproduce (27) we must have $g(y)=1$, which, as we show in the next section, is true in terms of appropriate variables. This is a self-consistency condition for the effective exponent laws. We will now verify much of the above perturbatively.

4. THE UNIVERSAL ONE-LOOP EQUATION OF STATE

To obtain a universal one-loop equation of state we need to make two demands: First that

$$H=\exp\left(\int_1^M \delta_{\text{eff}}\frac{dM'}{M'}\right)$$

when $t=0$ and second that for $H=0$ the equation of state has a zero at $x=-1$, where x is the scaling field introduced in Section 3. These demands ensure the effective exponent laws (24) and (27). Using

$$\bar{\lambda}(\rho)=\exp\left[\int_1^\rho (d-d_{\text{eff}})\frac{dx}{x}\right]$$

in a one-loop approximation and setting $\kappa=1$ for convenience, one finds for $t=0$ that $\delta_{\text{eff}}=(d_{\text{eff}}+2)/(d_{\text{eff}}-2)$.

Now consider the case when $t\neq 0$; our task is to get our expressions into the universal form (31). We define a variable $x=[a+b(y)\bar{\lambda}(y)]\,t(\rho)/\rho^2\kappa^2$ and choose a and b such that for $H=0$, $x=-1$ is a zero of the equation of state. Comparing powers of $\bar{\lambda}$ determines a and b. Substituting back into the equation of state and reexpressing the resulting expressions in terms of the coupling $h=a_1\bar{\lambda}$ gives

$$\begin{aligned}\mathscr{G}(x, y)=1+x&-\frac{[2/(5-d)(3-d)]\,h(y)}{\sum_{-\infty}^{\infty}[1+(2\pi n/y)^2]^{(d-7)/2}}\\&\times\sum_{-\infty}^{\infty}\left\{(1+x)\left[1+\left(\frac{2\pi n}{y}\right)^2\right]^{(d-3)/2}\right.\\&\left.-x\left[\frac{2}{3}+\left(\frac{2\pi n}{y}\right)^2\right]^{(d-3)/2}-\left[1+\frac{x}{3}+\left(\frac{2\pi n}{y}\right)^2\right]^{(d-3)/2}\right\}\end{aligned} \tag{33}$$

The universal form of the equation of state to one loop in terms of the two scaling fields x and y is then given by

$$H=\left[\exp\int_1^M\left(\frac{d_{\rm eff}+2}{d_{\rm eff}-2}\right)\frac{dM'}{M'}\right]\mathscr{G}(x,y)$$

in accordance with (31). Note that an essentially equivalent expression is obtained in expanding about the floating fixed point, the quantities in (33) being replaced by their floating fixed-point values to this order.

For $d=4$ in terms of the floating fixed point we have

$$\begin{aligned}\mathscr{G}(x,y)=1+x+&\frac{\sum_{-\infty}^{\infty}[1-2(2\pi n/y)^2][1+(2\pi n/y)^2]^{-5/2}}{\{\sum_{-\infty}^{\infty}[1+(2\pi n/y)^2]^{-3/2}\}^2}\\&\times\sum_{-\infty}^{\infty}\left\{(1+x)\left[1+\left(\frac{2\pi n}{y}\right)^2\right]^{1/2}-x\left[\frac{2}{3}+\left(\frac{2\pi n}{y}\right)^2\right]^{1/2}\right.\\&\left.-\left[1+\frac{x}{3}+\left(\frac{2\pi n}{y}\right)^2\right]^{1/2}\right\}\end{aligned}\tag{34}$$

For $d=3$ care should be taken in taking the limit; we are fortunate, however, in that the sums in this case can be reduced to elementary functions. One finds

$$\begin{aligned}\mathscr{G}(x,y)=1+x+&\frac{2h(y)\tanh(y/2)}{y(1+y/\sinh y)}\\&\times\left\{(1+x)\ln\sinh\left(\frac{y}{2}\right)-\ln\sinh\left[\frac{y}{2}\left(1+\frac{x}{3}\right)^{1/2}\right]\right.\\&\left.-x\ln\sinh\left[\frac{y}{2}\left(\frac{2}{3}\right)^{1/2}\right]\right\}\end{aligned}\tag{35}$$

The running coupling result is independent of the value of the initial coupling if it is set arbitrarily but finite at $\rho=\infty$; the resulting running coupling is then $h(y)=1+y/\sinh(y)$, which can easily be verified to be a solution of (15), in which

$$\varepsilon=1+\frac{y^2\coth(y/2)}{\sinh(y)+y}$$

The corresponding running and floating fixed-point forms of the equation of state are obtained by replacing $h(y)$ in (35) by these expressions, respectively. For the limit $L/\xi\to\infty$ one can also perform an ε expansion to derive the well-known results.[17,21] Finally we note that the coexistence curve is

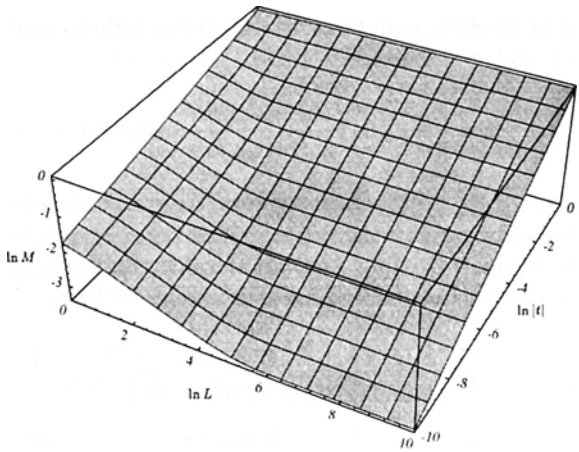

Fig. 1. The coexistence surface. Here ln M is plotted versus ln L and $\ln|t|$.

given by $x = -1$, which is a zero of $\mathscr{G}(x, y)$. This corresponds to a surface in the crossover case since an additional variable L enters. We present this surface in graphical form in Fig. 1. Figure 2 shows the coexistence curves for a set of different fixed layer thickness. We have separated the curves by choosing $T_c(L) = 1 - 1/(2 + L^{2.43})$, where $t = T_c(L) - T$. This specifies the value of T for which t intercepts the $M = 0$ axis, but does not affect the shape of the curves. The dependence of $T_c(L)$ on L is not determined in the above; we have therefore chosen it arbitrarily. Our prescription was to

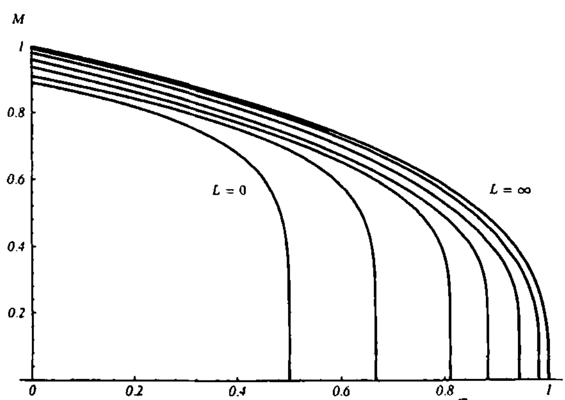

Fig. 2. The magnetization as a function of t for $L = 0$, 1, 1.65, 2.15, $L = 3$, $L = 5$ and $L = \infty$. We have separated the curves by choosing our shift in the form $t = 1 - 1/(2 + L^{2.43}) - T$.

choose $T_c(0)$ to be the value of the critical temperature given by the exact solution of the $2d$ Ising model and the value for $L=\infty$ to be 1/2. The functional form was taken to be $T_c(L)=1-1/(a+L^{1/\nu})$, $1/\nu$ to one loop being 2.43. The resulting graph compares favorably with the numerical results of Binder,[9] Fig. 25.

5. EFFECTIVE EXPONENTS TO ONE LOOP

In this section we will derive expressions for δ_{eff} and β_{eff} to one loop. From (23), noting that $\gamma_\varphi=0$ to one loop we have

$$\delta_{\text{eff}}=\frac{d\ln H}{d\ln M}=\left(\frac{d_{\text{eff}}(\rho)+2}{d_{\text{eff}}(\rho)-2}\right) \tag{36}$$

Working in terms of the floating fixed point and absorbing correction to scaling factors into redefinitions of H and M, we find that δ_{eff} becomes

$$\delta^*_{\text{eff}}=\left(\frac{d^*_{\text{eff}}(\rho)+2}{d^*_{\text{eff}}(\rho)-2}\right)=\left(\frac{6-\varepsilon(L\kappa\rho)}{2-\varepsilon(L\kappa\rho)}\right)=3+\varepsilon(L\kappa\rho) \tag{37}$$

where ρ is the solution of $\lambda(\rho)\,M^2(\rho)/2=\rho^2\kappa^2$ and we have expanded the denominator in $\varepsilon(L\kappa\rho)$. This is necessary, as we are implementing perturbation theory in terms of the floating fixed point. At the floating fixed point one obtains

$$(\rho\kappa)^{d-2}=\frac{\varepsilon(L\kappa\rho)}{a_1(L\kappa\rho)}\,M^2(\rho) \tag{38}$$

This transcendental equation must be solved for ρ and the solution substituted into

$$\delta^*_{\text{eff}}=8-d-(7-d)\,\frac{\sum_{n=-\infty}^{\infty}(4\pi^2n^2/\rho^2L^2\kappa^2)(1+4\pi^2n^2/\rho^2L^2\kappa^2)^{(d-9)/2}}{\sum_{n=-\infty}^{\infty}(1+4\pi^2n^2/\rho^2L^2\kappa^2)^{(d-7)/2}} \tag{39}$$

For $d=3$

$$\delta^*_{\text{eff}}=4+\frac{L^2\kappa^2\rho^2\coth(L\kappa\rho/2)}{\sinh L\kappa\rho+L\kappa\rho} \tag{40}$$

Denoting the solution of (38) as $L\kappa\rho=g(LM^{2/(d-2)})$ gives for $d=4$

$$\delta^*_{\text{eff}}=4-3\,\frac{\sum_{n=-\infty}^{\infty}(4\pi n^2/g^2)(1+4\pi^2n^2/g^2)^{-5/2}}{\sum_{n=-\infty}^{\infty}(1+4\pi^2n^2/g^2)^{-3/2}} \tag{41}$$

As $LM^{2/(d-2)}\to 0$, $\delta_{\text{eff}}\to 4$, and as $LM^{2/(d-2)}\to\infty$, $\delta_{\text{eff}}\to 3$.

Turning now to β_{eff}, to one loop it is

$$\beta_{\text{eff}} = \frac{d \ln M}{d \ln |t|} = \frac{\nu_{\text{eff}}}{2} (d_{\text{eff}} - 2) \tag{42}$$

Once again working in terms of the floating fixed point and absorbing corrections to scaling into redefinitions of t and M, we find that (42) becomes

$$\beta^*_{\text{eff}} = \frac{1}{2} - \frac{\varepsilon(L\kappa\rho)}{6} \tag{43}$$

To find ρ we need to solve $|t(\rho)| = \rho^2\kappa^2$. To lowest order it gives $\rho\kappa = |t|^{1/2}$, thus

$$\beta^*_{\text{eff}} = \frac{d-2}{6} + \frac{(7-d)}{6} \frac{\sum_{n=-\infty}^{\infty} (4\pi^2 n^2/L^2|t|)(1 + 4\pi^2 n^2/L^2|t|)^{(d-9)/2}}{\sum_{n=-\infty}^{\infty} (1 + 4\pi^2 n^2/L^2|t|)^{(d-7)/2}} \tag{44}$$

For $d = 3$

$$\begin{aligned} \beta^*_{\text{eff}} &= \frac{1}{6} + \frac{2}{3} \frac{\sum_{n=-\infty}^{\infty} (4\pi^2 n^2/L^2|t|)(1 + 4\pi^2 n^2/L^2|t|)^{-3}}{\sum_{n=-\infty}^{\infty} (1 + 4\pi^2 n^2/L^2|t|)^{-2}} \\ &= \frac{1}{3} - \frac{1}{6} \frac{L^2|t| \coth(L|t|^{1/2}/2)}{\sinh L|t|^{1/2} + L|t|^{1/2}} \end{aligned} \tag{45}$$

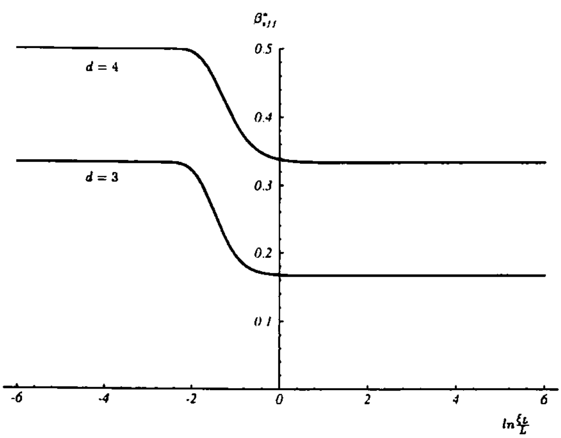

Fig. 3. The effective exponent β^*_{eff} for the four-dimensional layered geometry ($d = 4$) and three-dimensional layered geometry ($d = 3$) vs. $\ln(\xi_{Lt}/L)$. The exponent β^*_{eff} exhibits a crossover from the asymptotic value of 0.5 for small ξ_{Lt}/L to 0.33 for large ξ_{Lt}/L in the $d = 4$ case and from 0.33 to 0.17 in the $d = 3$ case.

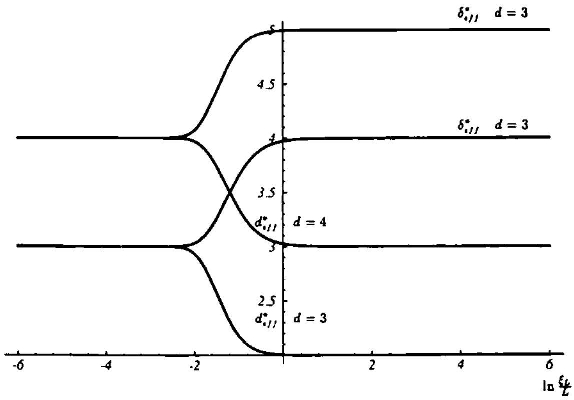

Fig. 4. The effective exponents δ^*_{eff} and d^*_{eff} for the four-dimensional ($d=4$) and three-dimensional ($d=3$) layered geometry vs. $\ln(\xi_{LM}/L)$. The exponent δ^*_{eff} exhibits a crossover from 4.0 to 3.0 in the $d=4$ case and from 5.0 to 4.0 in the $d=3$ case, while d^*_{eff} exhibits a crossover from the asymptotic value of 4.0 to 3.0 in the $d=4$ case and from 3.0 to 2.0 in the $d=3$ case, as ξ_{LM}/L ranges from small to large values.

Obviously, working with β^*_{eff} is much simpler than δ^*_{eff} because the condition determining ρ is much more amenable to a perturbative solution than that for δ^*_{eff}. We present β^*_{eff} in Fig. 3 in graphical form for 4-dimensional to 3-dimensional and 3-dimensional to 2-dimensional crossovers. Figure 4 presents δ^*_{eff} and d^*_{eff} similarly in graphical form for these crossovers.

One might enquire as to why the usual condition $M(\rho) = (\rho\kappa)^{d/2-1}$ was not used. The reason why it cannot be used is that it leads to an ill-defined perturbation theory in the limit $LM^{2/(d-2)} \to 0$ because setting a condition on M does not keep away from the critical region if λ can become very small. This cannot happen in the noncrossover case, but does happen here.

6. CONCLUSION

Previously[1] we had set out a formulation of how to treat perturbatively the crossover above T_c for a finite-size system, wherein the finite system itself could exhibit critical behavior. The present paper is a natural extension of this formulation to below T_c. The canonical problem to a large extent from the crossover point of view is the same either above or below T_c in that one would like an RG that "coarse grains" the effective degrees of freedom in an L-dependent way, as one knows that the physics is very L dependent. The natural consequence of an L-dependent RG is seen to be L-dependent anomalous dimensions and the appearance of ξ_L as the most natural scaling field in the problem as opposed to the bulk correlation length. We identified three such scaling fields that were capable of spanning the crossover: ξ_{Lt}, ξ_{LM}, and ξ_{LMt}. The first two represent physically the correlation length in finite-size systems above $T_c(L)$ in zero magnetic field and at $T = T_c(L)$, respectively. ξ_{LMt} is the true correlation length in the real physical system. For the crossover in question, however, all three are equally good nonlinear scaling fields. The L-dependent RG shows how correlation functions and particularly the equation of state can be written in a natural scaling form in terms of these scaling fields.

We have defined natural analogs of the critical exponents δ and β for the crossover and showed that these effective exponents satisfy the scaling laws $\delta_{\text{eff}} = (d_{\text{eff}} + 2 - \eta_{\text{eff}})/(d_{\text{eff}} - 2 + \eta_{\text{eff}})$ and $\beta_{\text{eff}} = \frac{1}{2}\nu_{\text{eff}}(d_{\text{eff}} - 2 + \eta_{\text{eff}})$, which are the analogs of the standard relations for the noncrossover case. These were the natural extension of the scaling law $\gamma_{\text{eff}} = \nu_{\text{eff}}(2 - \eta_{\text{eff}})$ derived in ref. 18. One subtlety was the appearance of an effective dimensionality d_{eff} in these relations. This object was seen to appear naturally as a representation of the fact that the scaling dimension of the operator φ^4 and hence the coupling constant λ changed across the crossover. In the noncrossover case γ_λ plays a rather minor role, for instance, representing the slowly varying and nonsingular corrections to scaling about the Wilson–Fisher fixed point. However, in the crossover case the change in degree of relevance of the φ^4 operator is very important and must be accounted for, and d_{eff} does this in a very natural fashion. It also appears very naturally if one thinks of it in the context of universality. The universality class of a system is specified by space dimensionality and symmetry. Here we interpolate between two universality classes with different space dimensions, hence it is quite natural to have a generalized universality in the sense that only γ_{φ^2}, γ_φ, and d_{eff} are required for a complete description. The effective exponents themselves are also universal quantities. The scaling fields for the crossover were shown to have a very natural representation in terms of the effective exponents and interpolated between just the ones one would expect in the asymptotic regimes. Having determined a universal form for the equation of state, we proceeded to determine it explicitly perturbatively. By implementing the effective exponent scaling laws one could determine the variable redefinitions necessary in order to make the equation universal. The equation of the crossover coexistence curve was determined.

There are several problems worth considering which stem directly from the considerations herein. First and foremost is the question of the discontinuity fixed point at the end of the coexistence curve. This fixed point cannot be seen in any of the expressions we derived here because the parameter that induces the crossover has not been included in the renormalization prescription, and therefore ones RG will be independent of it and hence the crossover will not be seen. In the case of the strong-coupling fixed point the natural thing to do is to implement an M-dependent renormalization, hence ones anomalous dimensions, etc., would all be explicitly M dependent. We will return to this issue in a future publication. Related to this is the question of the behavior below $T_c(L)$ of an $O(N)$ model, in particular the nonlinear σ model. Once again we will return to this issue in the future.

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